

MECHANICS OF MATERIALS

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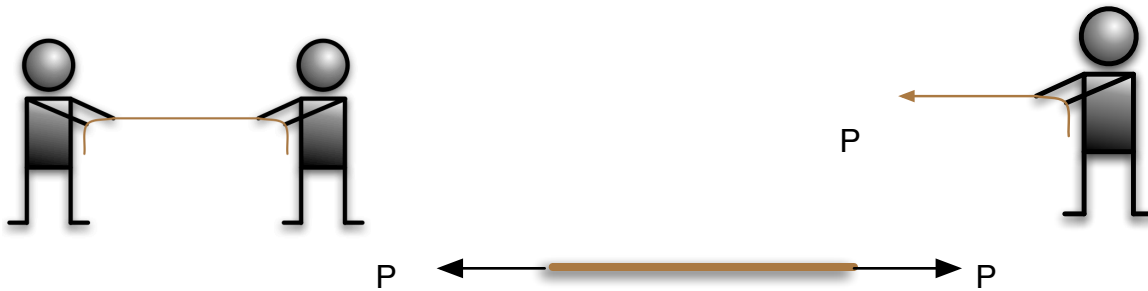
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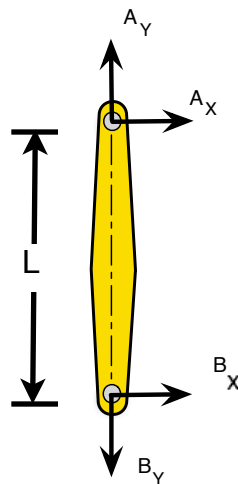
Uniaxial Forces and Stresses

The most fundamental of all loading conditions is of two co-linear forces applied to a straight rod or beam. A beam is a structural member that can resist bending whereas a rope has little bending resistance. The rope below is pulled on and resisted by the force P .



P is referred to as an axial force. Only two forces are required to keep the rope in equilibrium. The two forces in the Free Body Diagram, FBD, are collinear. A FBD of a member with two frictionless pins is used to prove that only two equal and opposite forces are required to satisfy equilibrium.

$$\begin{aligned}\sum F_y = 0 &= A_y - B_y \\ A_y &= -B_y \\ \sum F_x = 0 &= A_x + B_x \\ A_x &= -B_x \\ \sum M_A = 0 &= L B_x \\ A_x &= -B_x = 0\end{aligned}$$



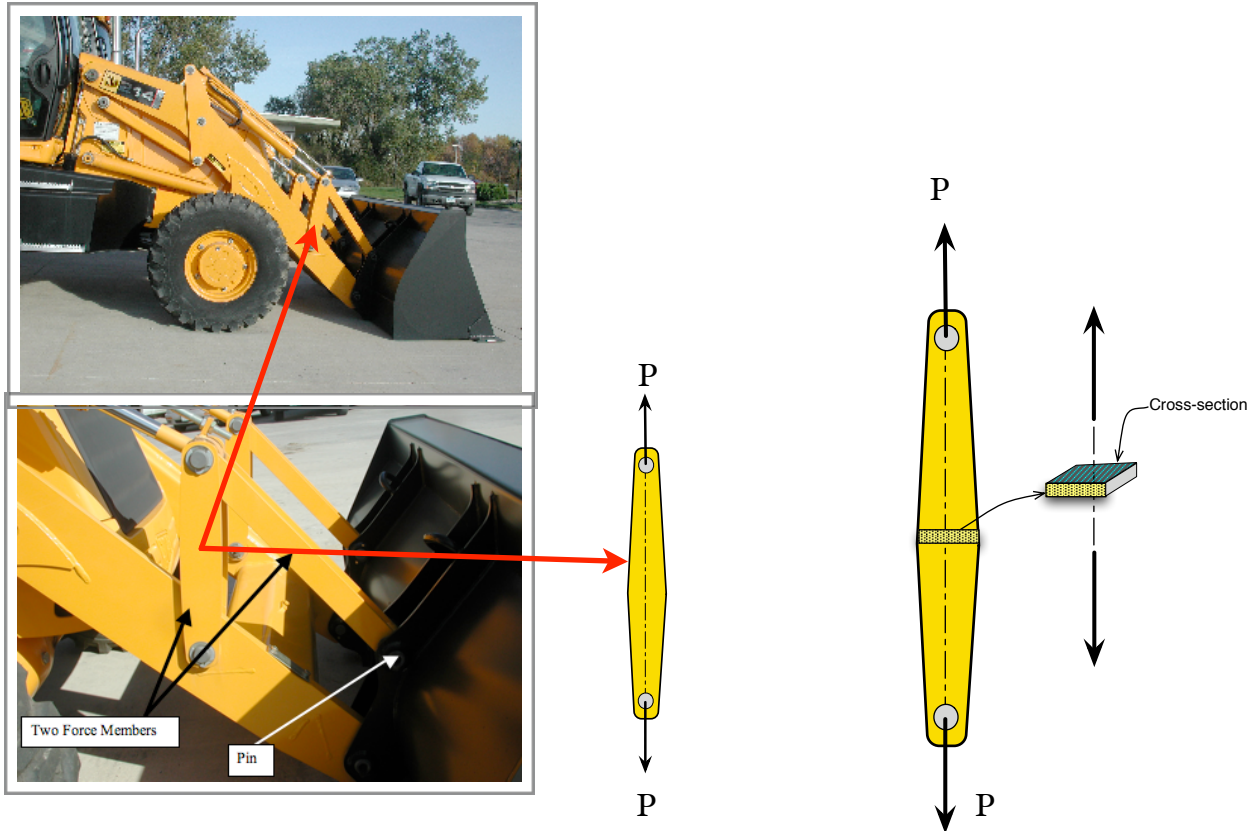
Note: A two force member has **no moment** applied at any location on the member, otherwise the two horizontal forces would not be zero.

The two horizontal forces are zero leaving only the two equal forces A_y and B_y . The forces are collinear.

TOOLBOX: As one of the tools in your toolbox, the inspection of a structure for two force members is an important one.

In the photograph below two members of a front end loader are identified as two force members. The line of action of the loads passes through the pins and centroids of all cross sections.

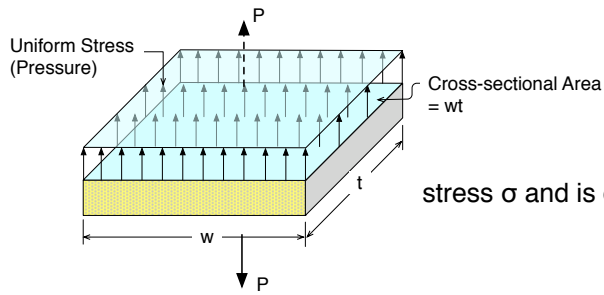
Since the vertical member is straight the load P passes through the centroids of the cross-sectional area at any position along the length of the member. This is a condition necessary to eliminate bending. The member will only elongate.



The force P acts on a typical cross-section shown on the right. The action P acting on the cross-sectional area needs to be characterized. The following observations are important.

- The material is not “stressed” as much at locations where the cross-sectional area is large.
- Small cross-sections are more likely to fail than larger sections.
- It is assumed that the cross section reacts uniformly to P .
- To account for the area variation, P is divided by the area A ; there will be more about this later.
- P/A is stress and has units of force/area.
- The strict assumption is that the member is straight and the cross-sectional area is constant along the length. This assumption is relaxed in practical situations where the change in cross-section is gradual. Large tapers will not have a uniform stress distribution within the cross-section. The member above has a small taper.
- The internal force passes through the centroid of the cross-section. This is done to separate stresses due to axial loads from bending moments.

ILLUSTRATION OF STRESS DISTRIBUTION



The force P is evenly spread over the cross-sectional area wt . The intensity of the force is the stress σ and is defined as force divided by the area.

$$\sigma = \frac{P}{A}$$

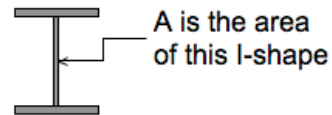
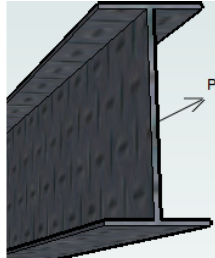
$$= \frac{P}{wt}$$

The stress is normal, meaning perpendicular, to the cross-sectional area and is often designated as a “normal stress.” Typical units are:

ENGLISH	ENGLISH	SI	SI
lb/in ²	psi	N/m ²	Pa
1000 psi	ksi	1000 Pa	kPa
(10 ⁶) psi	Msi	10 ⁶ Pa	MPa
		10 ⁹ Pa	GPa

EXAMPLE 1

I-beams such as the one represented below are commonly used in bridges and buildings. The axial load acting through the centroid of the cross-section causes the same type of stress as calculated by equation above.



An I-beam has a cross-sectional area of 5 in^2 and is subjected to a load of 20,000 lb [also referred to as 20 kip load where kip = 1000 lb]. What is the normal stress?

$$\begin{aligned}\sigma &= \frac{P}{A} \\ &= \frac{20,000 \text{ lb}}{5 \text{ in}^2} \\ &= \underline{\underline{4000 \text{ psi}}}\end{aligned}$$

Similarly 20,000 lb is also written as 20 kip.

$$\begin{aligned}\sigma &= \frac{P}{A} \\ &= \frac{20 \text{ kip}}{5 \text{ in}^2} \\ &= \underline{\underline{4 \text{ ksi}}}\end{aligned}$$

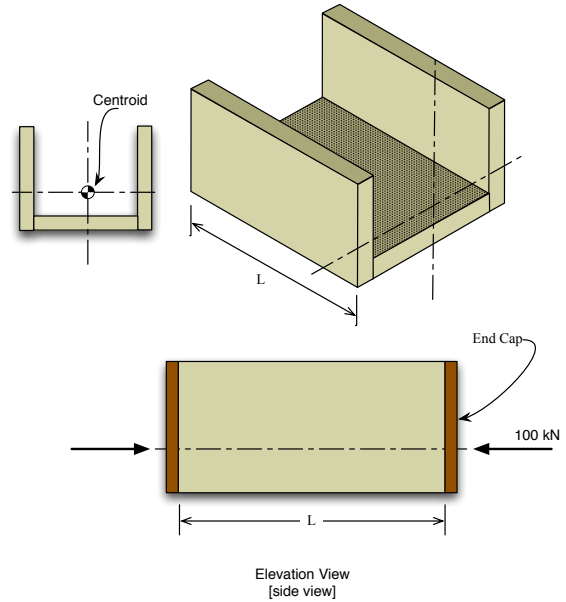
EXAMPLE 2

A C-channel must be used to support a load of 100 kN. The stress cannot exceed 150 MPa. What is the minimum area required to support the load?

$$\sigma = \frac{P}{A}$$

$$A = \frac{P}{\sigma}$$

$$\begin{aligned} &= \frac{100(10)^3 \text{ N}}{150(10)^6 \text{ N/m}^2} \\ &= 0.66\bar{6}(10)^{-3} \text{ m}^2 \\ &= \underline{\underline{667 \text{ mm}^2}} \end{aligned}$$

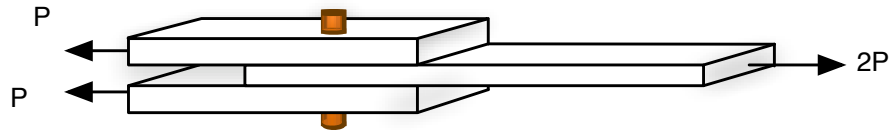


Notes:

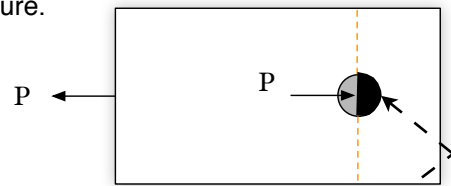
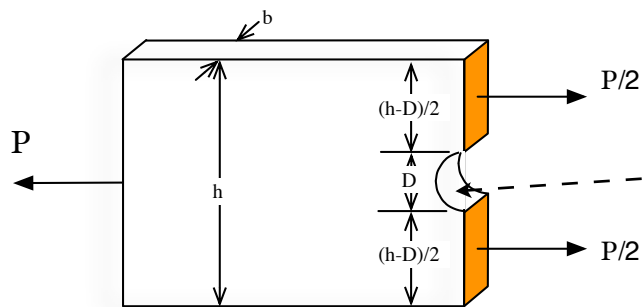
- The area is the same regardless of the shape (pipe, I-beam, rectangular, box beam, angle, etc.) .
- This is the minimum area required to satisfy the stated conditions.
- The uniaxial load must be applied through the centroid of the cross-section.
 - The applied load is distributed to the channel through end plates.
 - The load can then be applied at the centroid of the C-channel.

FAILURE AT MINIMUM CROSS-SECTION

The double lap joint at the right will be used to illustrate the identification of stresses at possible failure sites. A pin (bolt) is used to carry the load from the two outer plates to the middle plate.



Looking at the top and bottom plates, the reduced area of the plate at the pin (bolt) location can be a location of failure. A FBD is drawn that shows the possible **failure surface**.



Comments on the how the pin loads the hole:

- The pin pushes on the front portion of the hole.
- The portion of the hole shown is not loaded, thus the **failure surface** supports all of the load.

A force of $P/2$ acts on an area of

$$A = \frac{h-d}{2}b$$

thus,

$$\sigma = \frac{P}{A}$$

$$= \frac{P/2}{\frac{h-d}{2}b}$$

$$\sigma = \frac{P}{(h-d)b}$$

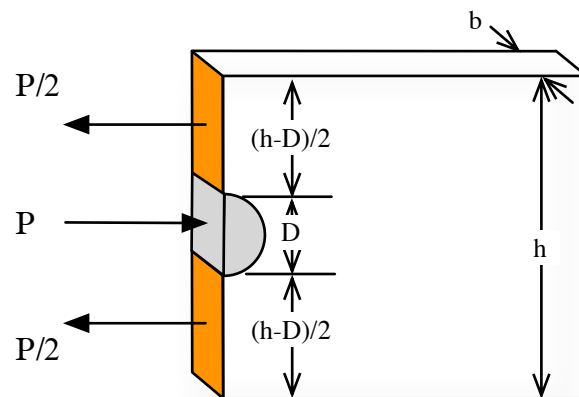
A second way of looking at the problem is to realize that the load P is being carried by all of the shaded **failure surface** with an area of

$$A = (h - d)b$$

$$\begin{aligned}\sigma &= \frac{P}{A} \\ &= \frac{P}{(h - d)b}\end{aligned}$$

giving the same answer as before.

The third way of looking at this problem is to use the FBD of the right portion of the plate. The same areas and forces are encountered in this case as in the other two approaches.



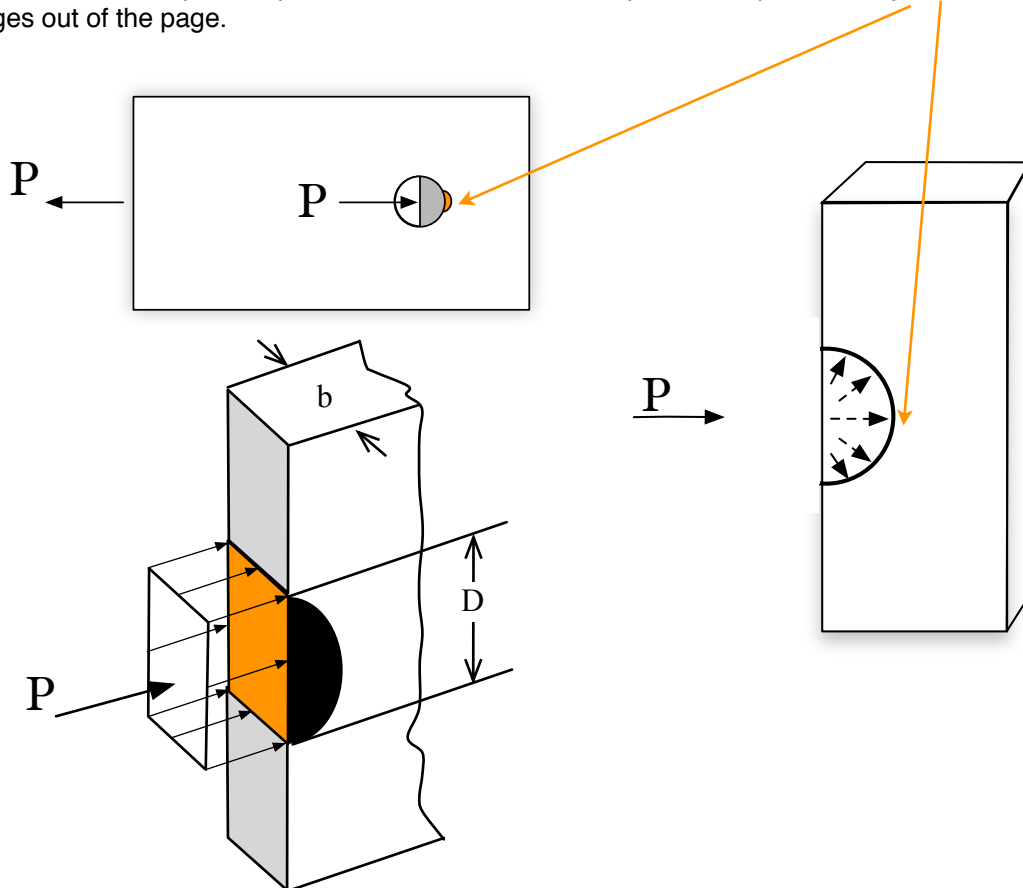
It is imperative that a FBD be drawn that cuts through the candidate failure surfaces. The areas and the forces acting on the surfaces are used to calculate the stresses.

$$A = (h - D) b$$

$$\sigma = \frac{P}{(h - D) b}$$

FAILURE DUE TO CONTACT FORCE: BEARING STRESS

The pin is pushing on the half arc. The pressure (stress) distribution is not uniformly distributed on the half arc surface. The distribution is dependent on the fit of the pin in the hole. A loose fit means that the stress will be larger because there will be less contact area. The stresses are also dependent on the materials used for the pin and plate. The area in front of the pin can be permanently deformed and material bulges out of the page.



A practical approach has historically been taken to characterize this type of problem. The bearing stress is calculated using the **bearing area** that is a rectangular patch size D by b . It is assumed that the bearing stresses are uniformly distributed over this area.

$$\sigma = \frac{P}{Db}$$

Experiments are run to obtain the failure stress using the above relationship. Designs are made based on the allowable (failure) stress and the calculated stress.

The skills required to be successful in using the bearing stress failure analysis are:

- Identify the contact area also called the bearing area.
- Draw the bearing area.
- Draw a good free body diagram with the **bearing area identified**.
- Use equilibrium to solve for the bearing stress.

Contact stresses are:

- Normal stresses
- Always compressive